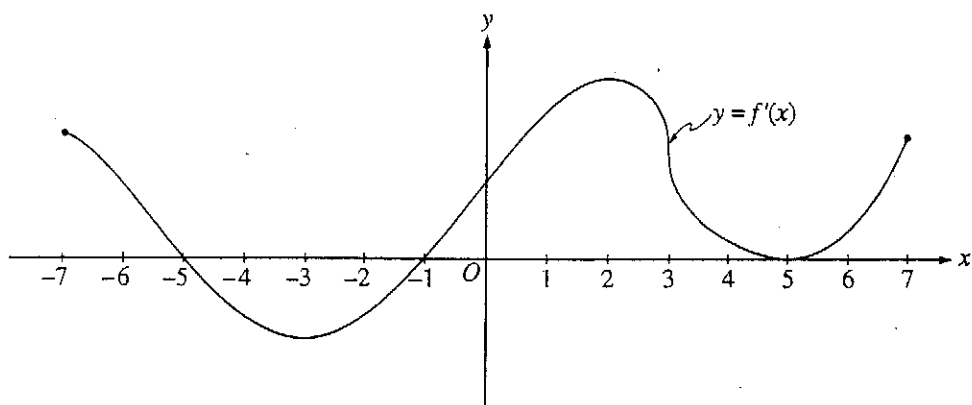


Topic 4 AB/BC

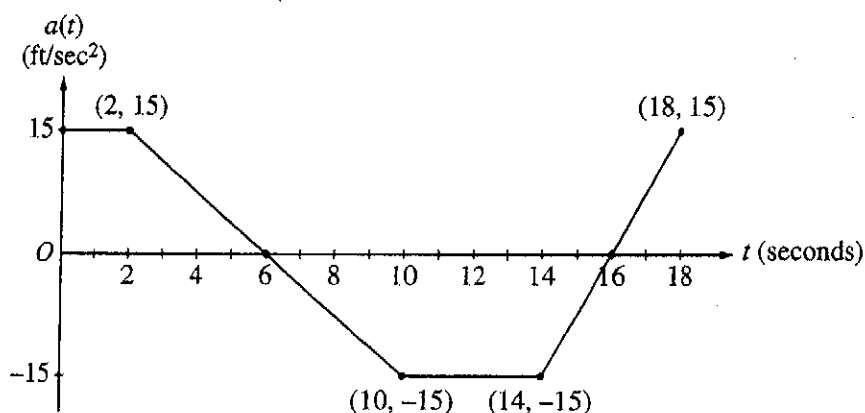
Information from
graph or about
graph, family of
functions f f' f''

2000 AP® CALCULUS AB FREE-RESPONSE QUESTIONS



3. The figure above shows the graph of f' , the derivative of the function f , for $-7 \leq x \leq 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.
- Find all values of x , for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.
 - Find all values of x , for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.
 - Find all values of x , for $-7 < x < 7$, at which $f''(x) < 0$.
 - At what value of x , for $-7 \leq x \leq 7$, does f attain its absolute maximum? Justify your answer.
-

2001 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS



3. A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \leq t \leq 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2 , is the piecewise linear function defined by the graph above.
- (a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?
 - (b) At what time in the interval $0 \leq t \leq 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?
 - (c) On the time interval $0 \leq t \leq 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.
 - (d) At what times in the interval $0 \leq t \leq 18$, if any, is the car's velocity equal to zero? Justify your answer.
-

END OF PART A OF SECTION II

4. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by

$$h'(x) = \frac{x^2 - 2}{x} \text{ for all } x \neq 0.$$

- 2001
- (a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
 - (b) On what intervals, if any, is the graph of h concave up? Justify your answer.
 - (c) Write an equation for the line tangent to the graph of h at $x = 4$.
 - (d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

2002 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

2. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{(t^2 - 24t + 160)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{(t^2 - 38t + 370)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \leq t \leq 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

- (a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round your answer to the nearest whole number.
- (b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.
- (c) Let $H(t) = \int_9^t (E(x) - L(x)) dx$ for $9 \leq t \leq 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$, and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park.
- (d) At what time t , for $9 \leq t \leq 23$, does the model predict that the number of people in the park is a maximum?
-

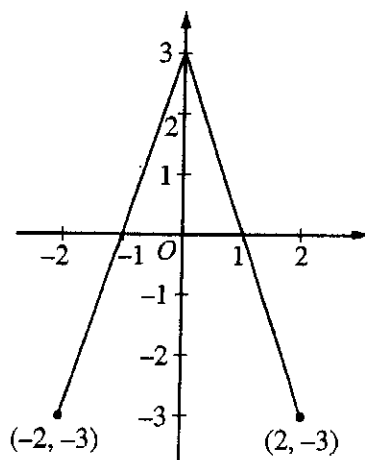
2002 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part B

Time—45 minutes

Number of problems—3

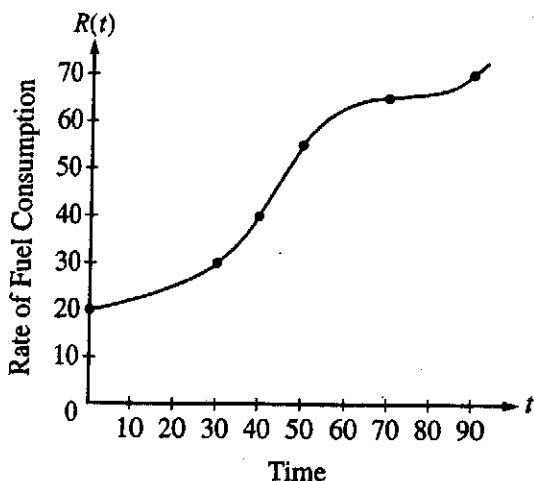
No calculator is allowed for these problems.



Graph of f

4. The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t) dt$.
- (a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.
 - (b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.
 - (c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.
 - (d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.
(Note: The axes are provided in the pink test booklet only.)

2003 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t . The graph of R and a table of selected values of $R(t)$, for the time interval $0 \leq t \leq 90$ minutes, are shown above.
- Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.
 - The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.
 - Approximate the value of $\int_0^{90} R(t) dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t) dt$? Explain your reasoning.
 - For $0 < b \leq 90$ minutes, explain the meaning of $\int_0^b R(t) dt$ in terms of fuel consumption for the plane.
Explain the meaning of $\frac{1}{b} \int_0^b R(t) dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

END OF PART A OF SECTION II

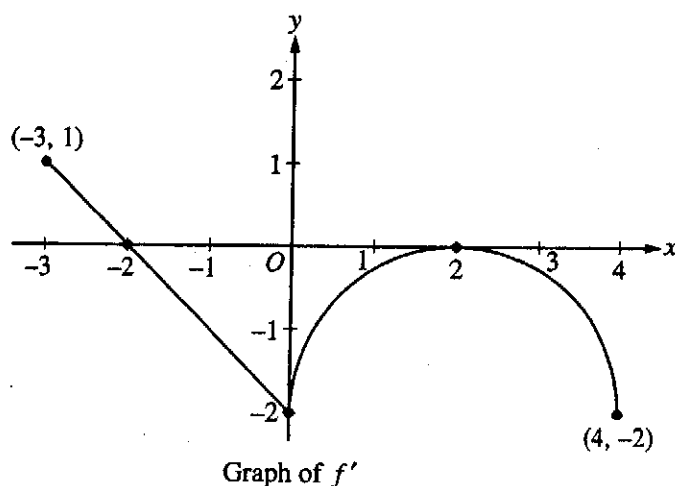
2003 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



4. Let f be a function defined on the closed interval $-3 \leq x \leq 4$ with $f(0) = 3$. The graph of f' , the derivative of f , consists of one line segment and a semicircle, as shown above.
- (a) On what intervals, if any, is f increasing? Justify your answer.
 - (b) Find the x -coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.
 - (c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.
 - (d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

2004 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

**CALCULUS AB
SECTION II, Part B**

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

4. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x -coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y -coordinate of P .

(c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P ? Justify your answer.

2005 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB SECTION II, Part B

Time—45 minutes

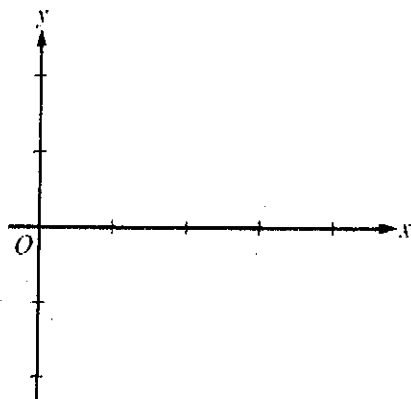
Number of problems—3

No calculator is allowed for these problems.

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

4. Let f be a function that is continuous on the interval $[0, 4]$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.
- (a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.
- (b) On the axes provided, sketch the graph of a function that has all the characteristics of f .

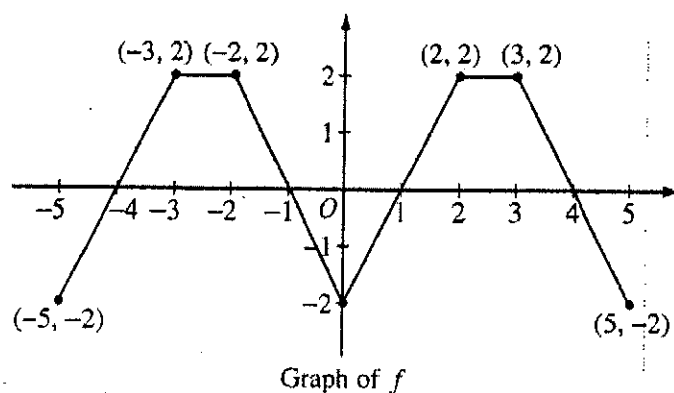
(Note: Use the axes provided in the pink test booklet.)



- (c) Let g be the function defined by $g(x) = \int_1^x f(t) dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.
- (d) For the function g defined in part (c), find all values of x , for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

WRITE ALL WORK IN THE TEST BOOKLET.

2006 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

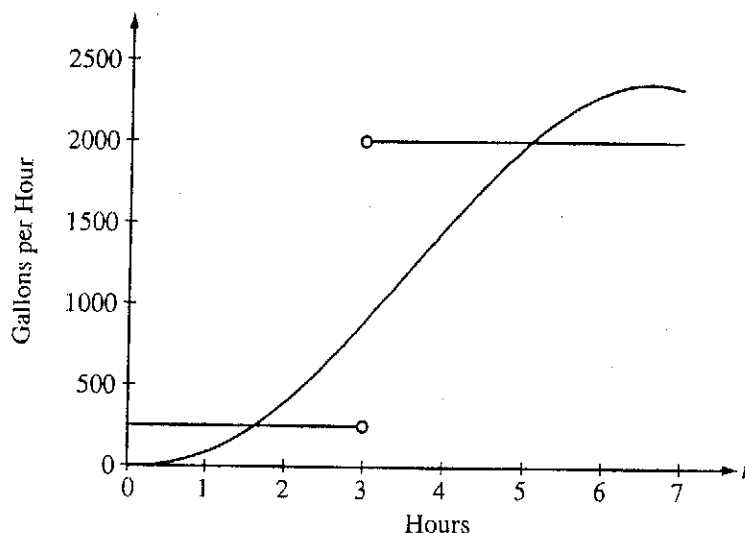


3. The graph of the function f shown above consists of six line segments. Let g be the function given by $g(x) = \int_0^x f(t) dt$.
- Find $g(4)$, $g'(4)$, and $g''(4)$.
 - Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.
 - Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f . Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of g at $x = 108$.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF PART A OF SECTION II

2007 AP® CALCULUS AB FREE-RESPONSE QUESTIONS



2. The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \leq t \leq 7$, where t is measured in hours. In this model, rates are given as follows:

- (i) The rate at which water enters the tank is $f(t) = 100t^2 \sin(\sqrt{t})$ gallons per hour for $0 \leq t \leq 7$.
 (ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$

The graphs of f and g , which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

- (a) How many gallons of water enter the tank during the time interval $0 \leq t \leq 7$? Round your answer to the nearest gallon.
 (b) For $0 \leq t \leq 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
 (c) For $0 \leq t \leq 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

2007 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

6. Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.
- (a) Find $f'(x)$ and $f''(x)$.
 - (b) For what value of the constant k does f have a critical point at $x = 1$? For this value of k , determine whether f has a relative minimum, relative maximum, or neither at $x = 1$. Justify your answer.
 - (c) For a certain value of the constant k , the graph of f has a point of inflection on the x -axis. Find this value of k .
-

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

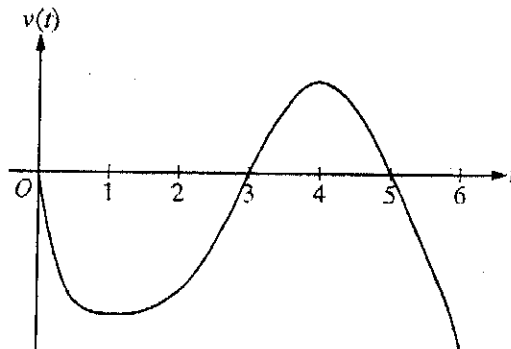
2008 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

**CALCULUS AB
SECTION II, Part B**

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of v

4. A particle moves along the x -axis so that its velocity at time t , for $0 \leq t \leq 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t -axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.
- (a) For $0 \leq t \leq 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.
 - (b) For how many values of t , where $0 \leq t \leq 6$, is the particle at $x = -8$? Explain your reasoning.
 - (c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.
 - (d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.
-

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

2008

6. Let f be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \frac{1 - \ln x}{x^2}$.

- (a) Write an equation for the line tangent to the graph of f at $x = e^2$.
 - (b) Find the x -coordinate of the critical point of f . Determine whether this point is a relative minimum, a relative maximum, or neither for the function f . Justify your answer.
 - (c) The graph of the function f has exactly one point of inflection. Find the x -coordinate of this point.
 - (d) Find $\lim_{x \rightarrow 0^+} f(x)$.
-

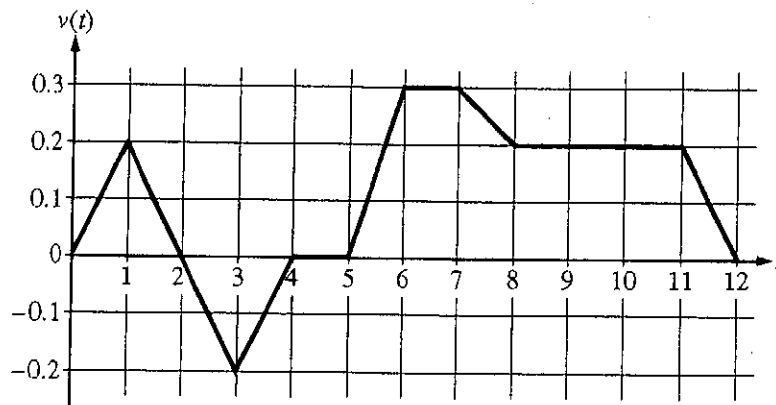
2009 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part A

Time—45 minutes

Number of problems—3

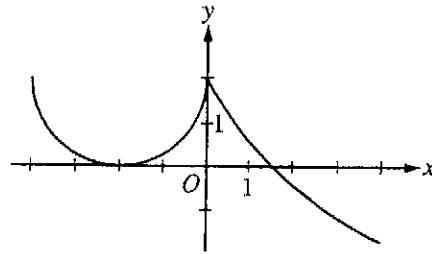
A graphing calculator is required for some problems or parts of problems.



1. Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \leq t \leq 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.
 - (a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.
 - (b) Using correct units, explain the meaning of $\int_0^{12} |v(t)| dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)| dt$.
 - (c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.
 - (d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15} \sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \leq t \leq 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

2009 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS



Graph of f'

6. The derivative of a function f is defined by $f'(x) = \begin{cases} g(x) & \text{for } -4 \leq x \leq 0 \\ 5e^{-x/3} - 3 & \text{for } 0 < x \leq 4 \end{cases}$

The graph of the continuous function f' , shown in the figure above, has x -intercepts at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \leq x \leq 0$ is a semicircle, and $f(0) = 5$.

- For $-4 < x < 4$, find all values of x at which the graph of f has a point of inflection. Justify your answer.
- Find $f(-4)$ and $f(4)$.
- For $-4 \leq x \leq 4$, find the value of x at which f has an absolute maximum. Justify your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM